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Effect of the unitary mixing scalar–vector.

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Abstract

We consider the procedure of dressing of scalar and vector particles when there exists the off-diagonal loop connecting vector and scalar propagators. Instead of single Dyson equations for scalar and vector, we have in this case a system of three equations for coupled full propagators. Using the $\pi - a_1$ system as an example, we discuss the physical meaning of this effect and the renormalization procedure for coupled propagators. The considered effect of the unitary S – V mixing may exists also in the "Higgs boson – Z (W)" system for extended electroweak models.

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1. Introduction.

The problem of the vector boson "dressing" (i.e. its turning into the finite width particle) was discussed last years in different aspects, see e.g. [1, 2, 3, 4]. It was initiated first of all by experiments on the W and Z production, where the more precise measurements allow to investigate rather subtle effects including radiative corrections. The main theoretical problems are related here with a gauge invariance, that is, how to introduce the mass and width of unstable gauge boson in a gauge-invariant manner.

At present work we discuss rather specific effect which can take place at dressing of a vector boson. It turns out that in some cases, when the vector current is not conserved, there appears a loop connecting vector and scalar propagators. It does not contradict with the angular momentum conservation because a closer look shows that such loop relates scalar propagator only with longitudinal part of vector propagator. As it is well-known, the longitudinal part corresponds to additional scalar degree of freedom which is contained in vector field. This degree of freedom is eliminated usually under quantization by some extra condition, for example, $\partial_\mu V^\mu = 0$. But with accounting of loops (i.e. quantum effects) this degree of freedom turns again into the "game" and it leads to a quite unusual form of full propagators. As a result, the joint efforts of scalar and the longitudinal part of vector propagator create an effective scalar resonance. In particular, such effect takes place in hadron physics in the system $\pi - a_1$ (or $\pi' - a_1$).

The considered effect of unitary S - V mixing can arise also for massive gauge bosons and in this case the Higgs scalars play the role of partners for W and Z in a joint dressing. But in the Standard Model, where only one scalar neutral Higgs particle exists, this phenomenon is absent, the corresponding off-diagonal loop is equal to zero. Nevertheless, this effect can arise in extended electroweak models, where charged or (and) pseudoscalar Higgs particles exist.

2. Formalism of S – V mixing.

Let us consider bare vector propagator in unitary gauge: ²

$$P^{\mu\nu} = i \frac{-g^{\mu\nu} + p^\mu p^\nu / M^2}{p^2 - M^2 + i \epsilon} = i \{ \rho_T^{\mu\nu} \cdot \pi_T(p^2) + \rho_L^{\mu\nu} \cdot \pi_L(p^2) \}, \quad (1)$$

where

$$\begin{aligned} \rho_T^{\mu\nu} &= -g^{\mu\nu} + p^\mu p^\nu / p^2, & \rho_L^{\mu\nu} &= p^\mu p^\nu / p^2 \\ \pi^T(p^2) &= \frac{1}{p^2 - M^2 + i \epsilon}, & \pi^L(p^2) &= \frac{1}{M^2}. \end{aligned} \quad (2)$$

To obtain a full propagator from a bare one, we should write the Dyson equation, which summarizes the self-energy inserts, and this equation determines the full propagator. But since vector propagator has tensor structure, transverse and longitudinal parts will dress in different ways. It is convenient to project the vector loop onto T and L components.

$$J^{\mu\nu} = \rho_T^{\mu\nu} \cdot J^T(p^2) + \rho_L^{\mu\nu} \cdot J^L(p^2) \quad (3)$$

Then we have the standard answer for full unrenormalized propagators:

$$\begin{aligned} \pi^T(p^2) = \frac{1}{p^2 - M^2 + i \epsilon} &\Rightarrow \Pi^T(p^2) = \frac{1}{p^2 - M^2 + J^T(p^2)} \\ \pi^L(p^2) = \frac{1}{M^2} &\Rightarrow \Pi^L(p^2) = \frac{1}{M^2 + J^L(p^2)} \end{aligned} \quad (4)$$

Another situation will arise if there exists the suitable by quantum numbers scalar particle, which dresses by the same intermediate state and, besides, there occurs the off-diagonal loop connecting the scalar and vector propagators. It is evidently from the beginning that such transition loop may relate a scalar propagator only with the longitudinal part of vector one. If the off-diagonal loop really exists then besides the full scalar and vector propagators a new object arises: a full off-diagonal propagator which relates the scalar and vector vertices.

²The unitary gauge is conventionally used in discussion of dressing and we also use it for simplicity. The S – V mixing may be considered in general ξ – gauge but there appears an additional scalar taking part in game – so called Higgs ghost.

As a result instead of one Dyson equation we have the system of three equations of the following form

$$\begin{aligned}\Pi_{11} &= \pi_{11} - \Pi_{11} J_{11} \pi_{11} - \Pi_{12}^\alpha i J_{21}^\alpha \pi_{11} \\ \Pi_{12}^\mu &= -\Pi_{11} i J_{12}^\alpha \pi_{22}^{\alpha\mu} - \Pi_{12}^\alpha J_{22}^{\alpha\beta} \pi_{22}^{\beta\mu} \\ \Pi_{22}^{\mu\nu} &= \pi_{22}^{\mu\nu} - \Pi_{21}^\mu i J_{12}^\alpha \pi_{22}^{\alpha\nu} - \Pi_{22}^{\mu\alpha} J_{22}^{\alpha\beta} \pi_{22}^{\beta\mu}.\end{aligned}\tag{5}$$

Here we introduced a matrix notations for scalar (11), vector (22), transition (12) propagators and similar for loops, the factor i in J_{12}^α is written for convenience. If $J_{12}^\alpha = 0$, we have the standard situation of single equations. For off-diagonal propagators and loops we can pass over to corresponding scalar functions.

$$\begin{aligned}\Pi_{12}^\alpha(p) &= p^\alpha \Pi_{12}(p^2), \quad J_{12}^\alpha(p) = p^\alpha J_{12}(p^2), \\ \Pi_{21}^\alpha(p) &= -\Pi_{12}^\alpha(p), \quad J_{21}^\alpha(p) = -J_{12}^\alpha(p)\end{aligned}\tag{6}$$

The last equation in the system (5) should be projected onto transverse and longitudinal components. As a result we obtain the single equation for transverse component

$$\Pi_{22}^T = \pi_{22}^T - \Pi_{22}^T J_{22}^T \pi_{22}^T\tag{7}$$

and the transverse part remains as before (4). As for the longitudinal component, it enters the system of three equations.

$$\begin{aligned}\Pi_{11} &= \pi_{11} - \Pi_{11} J_{11} \pi_{11} + p^2 \Pi_{12} i J_{21} \pi_{11} \\ \Pi_{12} &= -\Pi_{11} i J_{12} \pi_{22}^L - \Pi_{12} J_{22}^L \pi_{22}^L \\ \Pi_{22}^L &= \pi_{22}^L + p^2 \Pi_{21} i J_{12} \pi_{22}^L - \Pi_{22}^L J_{22}^L \pi_{22}^L\end{aligned}\tag{8}$$

Solution of this system in most general form is

$$\begin{aligned}\Pi_{11}(p^2) &= \frac{(\pi_{22}^L)^{-1} + J_{22}^L}{D(p^2)}, \quad \Pi_{12}(p^2) = -\frac{i J_{12}(p^2)}{D(p^2)}, \quad \Pi_{22}^L(p^2) = \frac{(\pi_{11})^{-1} + J_{11}}{D(p^2)}, \\ D(p^2) &= [(\pi_{11})^{-1} + J_{11}] [(\pi_{22}^L)^{-1} + J_{22}^L] - p^2 [J_{12}(p^2)]^2.\end{aligned}\tag{9}$$

Note that similar formulae ("resonances with unitary mixing") are well-known in hadron physics, see, e.g. [5], in application to scalar resonances. They provide the unitary condition in the case of few resonances with the same quantum numbers.

So with accounting of S-V mixing the dressed propagators take the form:

$$\Pi_{22}^T(p^2) = \frac{1}{p^2 - M^2 + J_{22}^T(p^2)}\tag{10}$$

$$\Pi_{11}(p^2) = \frac{M^2 + J_{22}^L(p^2)}{D(p^2)}, \quad \Pi_{12}(p^2) = -\frac{iJ_{12}(p^2)}{D(p^2)}, \quad \Pi_{22}^L(p^2) = \frac{p^2 - \mu^2 + J_{11}(p^2)}{D(p^2)}. \quad (11)$$

$$D(p^2) = [p^2 - \mu^2 + J_{11}(p^2)] [M^2 + J^L(p^2)] - p^2 [J_{12}(p^2)]^2 \quad (12)$$

Here M and μ are bare masses of vector and scalar particles.

The obtained formulae have rather clear meaning. The loop connecting scalar and vector propagator is in fact the transition of scalar particle again to scalar "particle" contained in vector field V^μ . For free vector field this degree of freedom is usually removed by some extra condition providing the self-consistent quantization. But with accounting of loop effects this degree of freedom appears again in the form of mixing of the longitudinal part of vector propagator with scalar one. The necessary condition for such effect is the non-conservation of the corresponding vector current $\partial_\mu J^\mu \neq 0$.

3. The $\pi - a_1$ system.

The described situation of the unitary $S - V$ mixing takes place in particular in the system of π and a_1 mesons which have couplings with the three-pion system. It is generally believed that 3π system is saturated by the quasi-two-particle states $\pi\sigma$ and $\pi\rho$. We shall consider below the simplest case of a joint π and a_1 dressing by $\pi\sigma$ state.

Let us define the vertices in momentum space.

$$\underline{a_1(P) \rightarrow \pi(k) \sigma(q)} \quad \text{vertex with vector index } \mu \text{ in diagram has the factor} \\ (-1) g_{a_1\pi\sigma} (k - q)^\mu,$$

$$\underline{\pi(P) \rightarrow \pi(k) \sigma(q)} \quad \text{vertex is} \quad i g_{\sigma\pi\pi}.$$

We shall present the results of calculations below, where μ and M are bare masses of π and a_1 mesons, and m is the σ -meson mass.

The transition loop $\pi - a_1$ differs from zero ³ and is of the form:

$$J_{12}^\mu(p) = i g_{a_1\pi\sigma} g_{\sigma\pi\pi} \int \frac{d^4 l}{(2\pi)^4} \frac{(2l - p)^\mu}{(l^2 - \mu^2)((l - p)^2 - m^2)} = p^\mu J_{12}(p^2), \quad (13)$$

³Note that more general form of $a_1\pi\sigma$ vertex (adding of total derivation term) can not lead to zero answer for transition loop. A nonzero contribution arises also from the $\pi\rho$ intermediate state.

where

$$J_{12}(p^2) = \frac{g_{a_1\pi\sigma} g_{\sigma\pi\pi}}{16\pi} (m^2 - \mu^2) \frac{1}{\pi} \int \frac{ds}{s(s-p^2)} \cdot \left(\frac{\lambda(s, m^2, \mu^2)}{s^2} \right)^{1/2}. \quad (14)$$

Here λ is the well-known function $\lambda(a, b, c) = (a - b - c)^2 - 4bc$, limits of integration here and below are not shown, they are evident.

Loop in the a_1 - propagator.

$$J_{22}^{\mu\nu}(p) = -i g_{a_1\pi\sigma}^2 \int \frac{d^4l}{(2\pi)^4} \frac{(2l-p)^\mu (2l-p)^\nu}{(l^2 - \mu^2)((l-p)^2 - m^2)} = g^{\mu\nu} \cdot A(p^2) + p^\mu p^\nu \cdot B(p^2) \quad (15)$$

Discontinuities of A and B functions are:

$$\begin{aligned} \Delta A(p^2) &= -i \frac{g_{a_1\pi\sigma}^2}{24\pi} p^2 \left(\frac{\lambda}{p^4} \right)^{3/2}, \\ \Delta B(p^2) &= i \frac{g_{a_1\pi\sigma}^2}{24\pi} \left[\frac{\lambda}{p^4} + \frac{3(m^2 - \mu^2)^2}{p^4} \right] \left(\frac{\lambda}{p^4} \right)^{1/2}, \\ \lambda &= \lambda(p^2, m^2, \mu^2). \end{aligned} \quad (16)$$

To restore the analytical function through its discontinuity, we need one subtraction for A and two subtractions for B. After that the transverse and longitudinal components are easily calculated. ⁴

$$J_{22}^T = -A(p^2), \quad J_{22}^L = A(p^2) + p^2 B(p^2) \quad (17)$$

That gives:

$$\begin{aligned} J_{22}^T(p^2) &= -A(0) - p^2 A'(0) + \frac{g_{a_1\pi\sigma}^2}{48\pi} \frac{p^4}{\pi} \int \frac{ds}{s(s-p^2)} \left(\frac{\lambda}{s^2} \right)^{3/2} \\ J_{22}^L(p^2) &= A(0) + p^2 [A'(0) + B(0)] + \frac{g_{a_1\pi\sigma}^2}{16\pi} (m^2 - \mu^2)^2 \frac{p^4}{\pi} \int \frac{ds}{s(s-p^2)} \frac{1}{s^2} \left(\frac{\lambda}{s^2} \right)^{1/2}. \end{aligned} \quad (18)$$

Loop in the π - propagator.

$$\begin{aligned} J_{11}(p^2) &= -i g_{\sigma\pi\pi}^2 \int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2 - \mu^2)((l-p)^2 - m^2)} \\ &= J_{11}(0) + \frac{g_{\sigma\pi\pi}^2}{16\pi} \cdot \frac{p^2}{\pi} \int \frac{ds}{s(s-p^2)} \left(\frac{\lambda(s, m^2, \mu^2)}{s^2} \right)^{1/2} \end{aligned} \quad (19)$$

⁴In such a way the poles $1/p^2$ in propagator are canceled automatically from the all following expressions.

One can see that the loop integrals J_{11} , J_{12} , J_{22}^L , involved into the system of equations may be expressed by the same function

$$H(p^2) = \frac{1}{\pi} \int \frac{ds}{s(s-p^2)} \left(\frac{\lambda(s, m^2, \mu^2)}{s^2} \right)^{1/2} \quad (20)$$

These loops have the form (with minimal subtractions)

$$\begin{aligned} J_{11}(p^2) &= J_{11}(0) + g_1^2 \cdot p^2 H(p^2) \\ J_{12}(p^2) &= g_1 g_2 \cdot H(p^2) \\ J_{22}^L(p^2) &= E + p^2 F + g_2^2 \cdot H(p^2), \end{aligned} \quad (21)$$

where we introduced the notations $g_1^2 = g_{\sigma\pi\pi}^2/16\pi$, $g_2^2 = (m^2 - \mu^2)^2 \cdot g_{a_1\pi\sigma}^2/16\pi$ and

$$E = A(0) - g_2^2 H(0), \quad F = A'(0) + B(0) - g_2^2 H'(0) \quad (22)$$

Here $H(0)$ and $H'(0)$ are known quantities.

Let us consider the function $D(p^2)$ in denominator of full propagators.

$$\begin{aligned} D(p^2) &= [p^2 - \mu^2 + J_{11}(p^2)] [M^2 + J_{22}^L(p^2)] - p^2 [J_{12}(p^2)]^2 = \\ &= [p^2 - \mu^2 + J_{11}(0)] [M^2 + E + p^2 F] + \\ &\quad + H(p^2) \{ p^2 g_1^2 (M^2 + E + p^2 F) + g_2^2 [p^2 - \mu^2 + J_{11}(0)] \} \end{aligned} \quad (23)$$

Note that products of loops are canceled. The similar cancelation arises for mixing of resonances with the same spin [5]. But if to consider a few intermediate states then such cancelation of all terms of the g^4 order takes place only at some relations between coupling constants.

Renormalization.

We shall renormalize propagators by means of the subtraction on the mass shell, supposing that in above formulae μ and M are the renormalized masses of pion and a_1 .

First of all let us renormalize the transverse part of vector propagator which has the Breit–Wigner form. Normalization upon the total width requires to subtract J_{22}^T twice on the mass shell.

$$Re J_{22}^T(M^2) = Re J_{22}^{T'}(M^2) = 0 \quad (24)$$

That defines the constants $A(0)$ and $A'(0)$.

In the case of few coupled propagators the procedure of subtraction on a mass shell becomes somewhat different. A zero of the function $D(p^2)$ gives rise to pole of the full pion propagator. If we require the unit residue of the renormalized pion propagator then from the first equation of the system (8) we have the manifest condition :

$$\Delta(p^2) \equiv \Pi_{11}(p^2) J_{11}(p^2) - p^2 \Pi_{12}(p^2) i J_{12}(p^2) = 0 \quad \text{at } p^2 = \mu^2. \quad (25)$$

It means that sum of all inserts into the external pion line is equal to zero. Using the solutions (11), one can obtain for the function $\Delta(p^2)$:

$$\Delta(p^2) = \frac{X(p^2)}{D(p^2)} = \frac{X(p^2)}{(p^2 - \mu^2)(M^2 + J_{22}^L(p^2)) + X(p^2)}, \quad (26)$$

where

$$X(p^2) = J_{11}(p^2) (M^2 + J_{22}^L(p^2)) - p^2 [J_{12}(p^2)]^2. \quad (27)$$

One can see from this expression that the requirement (25) is equivalent to the following conditions:

$$X(\mu^2) = X'(\mu^2) = 0. \quad (28)$$

In principal these requirements may be realized in different ways. But there exists the natural physical condition on the longitudinal part of vector propagator: it should not have the pion pole after renormalization. So in the following we shall accept this anzats for renormalization procedure:

$$\begin{aligned} & \textit{The longitudinal part of vector propagator} \\ & \textit{should not get a pole of scalar particle.} \end{aligned} \quad (29)$$

It leads to the following requirements for loops ⁵ (see (11),(28)):

$$J_{11}(\mu^2) = J'_{11}(\mu^2) = 0, \quad J_{12}(\mu^2) = 0 \quad (30)$$

As a result $D(p^2)$ in vicinity of $p^2 = \mu^2$ is of the form:

$$D(p^2) = (p^2 - \mu^2) (M^2 + J_{22}^L(p^2)) + O((p^2 - \mu^2)^2) \quad (31)$$

One can see from (11), (30) that only the full pion propagator Π_{11} has a pole $1/(p^2 - \mu^2)$, as for Π_{12} and Π_{22}^L – they do not have it.

⁵One can check by direct calculation that it is the only possibility to satisfy (28) at least at small coupling constant $g_{a_1 \pi \sigma}$.

Note that after the mass renormalization for a_1 and π the longitudinal loop is not defined completely.

$$J_{22}^L(p^2) = E + p^2 F + g_2^2 H(p^2) \quad (32)$$

Recall that here the parameter F is still arbitrary. But if in (32) $F \neq 0$ then $D(p^2)$ necessarily has a zero in complex plane at one of the Riemann sheets, and we have put this parameter to zero not to get poles in propagators besides the $p^2 = \mu^2$ one.

So we defined completely the full renormalized propagators with account of the loop S–V transitions. The final formulae are collected in Appendix.

4. The $\pi' - a_1$ system.

We shall consider a case when both partners are located higher the threshold, $\pi' - a_1$ system for shortness. We can use the above formulae with changing π to π' in propagators (but not in loops). First of all let us discuss the question: could not the longitudinal part obtain a scalar pole in complex plane? In other words, would not the anzats (29) contradictory in this case ?

Let the function $D(p^2)$ (12) now has pole in a complex plane at the second Riemann sheet $D(\mu_c^2) = 0$. The absence of this pole in the longitudinal part of vector propagator means that (11):

$$p^2 - \mu'^2 + J_{11}(p^2) = 0 \quad \text{at} \quad p^2 = \mu_c^2. \quad (33)$$

Then from the explicit form of $D(p^2)$ there follows that the off-diagonal loop is equal to zero as well.

$$J_{12}(p^2) = 0 \quad \text{at} \quad p^2 = \mu_c^2 \quad (34)$$

Let us write the more general expressions for loops as compared with (21), introducing arbitrary subtraction polynomials P_{ij} .

$$\begin{aligned} J_{11}(p^2) &= g_1^2(P_{11} + p^2 H(p^2)) \\ J_{12}(p^2) &= g_1 g_2(P_{12} + H(p^2)) \end{aligned} \quad (35)$$

We can obtain the condition of compatibility of (33) and (34), eliminating the function $H(p^2)$ from them.

$$p^2 - \mu'^2 + g_1^2(P_{11} - p^2 P_{12}) = 0 \quad \text{at} \quad p^2 = \mu_c^2 \quad (36)$$

This expression should be a second order over p^2 polynomial to have a zero in complex plane. So in this case the anzats (29) seems to be non- contradictory too, but higher order of polynomials are needed as compared with the $\pi - a_1$ case.

The amplitude $\pi\sigma \rightarrow (\pi', a_1) \rightarrow \pi\sigma$.

The calculation of this amplitude allows to look at manifestation of S-V mixing in matrix element. First of all it is instructive to write this amplitude with bare propagators of π' and a_1 in s-channel.

$$M^B = \frac{g_{A\pi\sigma}^2}{M^2 - s - i\epsilon} \cdot \frac{s(t-u) + (m^2 - \mu^2)^2}{s} - \frac{g_{A\pi\sigma}^2}{M^2} \cdot \frac{(m^2 - \mu^2)^2}{s} + \frac{g_{\pi'\pi\sigma}^2}{\mu^2 - s - i\epsilon}$$

Here the first term results from the transverse part of a_1 propagator, the second one from longitudinal part and third – from π' propagator. We can see from the angular dependence that first term has the angular momentum $J = 1$ in s-channel and the remaining ones have $J = 0$. So the longitudinal part of vector propagator gives rise to the background nonresonance contribution to the $J = 0$ amplitude. At the dressing of propagators there takes place a unitarization of the p- and s-wave amplitudes separately. As for the s-wave, there takes place a joint unitarization in the system "pole + background". One can suspect that with the full propagators (and with accounting the off-diagonal terms) we shall obtain the unitary expressions both for p- and s-wave amplitudes. It is easy to verify with using of our solutions (11) that this is the case (that's an additional check of correctness of calculations), but we do not present here these formulae. So in the amplitude $\pi\sigma \rightarrow \pi\sigma$ with $J = 0$ there takes place a unitarization of resonance with the presence of background, and the background is generated by the longitudinal part of vector propagator.

5. S – V mixing and gauge bosons.

Let us clarify a question whether this effect of the S-V mixing exists for gauge bosons. To this end we should look at the transition loop, connecting W (Z) with its

scalar partner – Higgs particle.

$$J_{12}^\alpha(p) \sim \int \frac{d^4 l}{(2\pi)^4} Sp \left\{ 1 \cdot \frac{i}{\hat{l} - \hat{p} - m_1} \gamma^\alpha (1 - a\gamma^5) \frac{i}{\hat{l} - m_2} \right\} \quad (37)$$

One can see that the γ^5 term does not contribute and result is proportional to $m_1 - m_2$, i.e. integral is equal to zero for fermion–antifermion loop. But if there is the pseudoscalar vertex instead of scalar one in (37), then such loop is nonzero with any fermion masses. It means that Z^0 boson can be connected by loop with a pseudoscalar particle, and W^\pm can transit into the charged scalar (pseudoscalar) Higgs particle. So in the Standard Model (one neutral scalar Higgs particle) there is no effect of unitary S–V mixing but in extended (super–extended) variants of electroweak models this effect can occur.

6. Discussion.

So the existence of loop S–V transitions leads to a system of Dyson equations for full propagators and the obtained full propagators have more complicated form. Another consequence – the appearance of unusual transition propagators, connecting the scalar and vector vertices. This phenomenon, unexpected at first sight, is in fact a very logical and has the transparent meaning. The reason is the simple fact that vector field has an additional $J = 0$ degree of freedom.

We considered in detail the renormalization of coupled propagators for the $\pi - a_1$ system, taking into account the $\pi\sigma$ intermediate state. That's not so real physical example but for coupled propagators the renormalization procedure is not so evident. In our opinion there exists the natural physical requirement for renormalization, namely, that the longitudinal part of vector propagator should not get the poles. In other words the corresponding degree of freedom remains unphysical after renormalization. This requirement defines completely the renormalization procedure.

Up to now we have not touched on the question whether the considered effect of unitary S – V mixing leads to some physical consequences besides the redefinition of parameters in $J = 0$ system. This question should be considered in more detail, here we shall only say few words from general point of view.

If speaking about effective scalar resonance (e.g. in $\pi' - a_1$ system) it's rather

difficult to imagine such situation, where the $S - V$ mixing will lead to observed effects. Indeed, from phenomenological point of view it gives a background spin 0 contribution which will be unitarized together with a pole. But in real life there exist different sources of background and to observe this effect we need some special situation: it should be rather broad resonance and background contributions from other sources should be small.

Some peculiar situation takes place in the case of π – meson which is located under threshold of any reaction. At first sight it seems that $S - V$ mixing gives only small correction of m_π^2/m_A^2 type to standard form of full π – meson propagator because of very different masses of mixed particles. But, firstly, there appeared the off-diagonal transitions which were absent in a standard picture. And secondly, let us remember that among the all π – meson interaction vertices there is one vertex with the special properties – that’s the electrodynamical one. So I suppose that it would be interesting to look at such π – meson processes from the point of view of the $S - V$ mixing manifestation. The set of such processes is well-known, see e.g. discussion of the off-shell π -meson effects in [6]

It’s possible that the effect under consideration takes place for gauge bosons W , Z but only in the case of nonstandard Higgs particles (charged or pseudoscalar). In particular the $S - V$ mixing can change the pattern of the CP-violating asymmetry between $t \rightarrow b\tau^+\nu_\tau$ and $\bar{t} \rightarrow \bar{b}\tau^-\bar{\nu}_\tau$, considered in [7, 4] (this asymmetry is proportional to imaginary part of longitudinal propagator). But recall that in framework of the Standard Model there is no such effect.

In principal the $S - V$ mixing may be considered in arbitrary ξ -gauge, where additional scalar ghost exists, but it will lead to more complicated picture. Nevertheless, the above formalism allows to look at the single vector meson dressing in general ξ -gauge from another side, at least it allows to clarify some used approximations.

In particular, at discussion in [8] it was used the general expression from [9] for the full nonrenormalized vector propagator of the W in general ξ -gauge. It turns out that this very complicated from first sight expression may be written just in form (11). That’s natural because in the system ” W – ghost” there arise the loop scalar-vector transitions. Then some simplifications have been made in [8], which in fact are the following: in function D (12) the term $(J_{12})^2$ is omitted since it is of the order of g^4 . Evidently, after that the formulae (11) become rather trivial. But when viewed closely,

such approximation looks as unreasonable. Indeed, in the function D there are other terms of the same g^4 order which are not omitted. Moreover we have seen that all terms of the g^4 order are canceled with each other in D . So the approximated formula for full vector propagator in arbitrary ξ -gauge from Ref. [8] looks as unjustified. But it turns out that in the limit $\xi \rightarrow \infty$ (unitary gauge) the exact answer is restored.

The similar phenomenon of unitary mixing of different spin particles (or fields) should be expected at dressing of spin 2 particle. It is described by the symmetry tensor $T^{\mu\nu}$ with 10 degrees of freedom and only 5 of them correspond to spin 2 particle. So at some conditions there can appear a loop tensor-scalar or tensor-vector transitions.

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Appendix.

We present here the final formulae for renormalized propagators in $\pi - a_1$ system.
The transverse part of vector propagator:

$$\begin{aligned}\Pi_{22}^T(p^2) &= \frac{1}{M^2 - p^2 + J_{22}^T(p^2)} \\ J_{22}^T(p^2) &= \frac{g_{a_1\pi\sigma}^2}{48\pi} \left[G(p^2) - \text{Re } G(M^2) - (p^2 - M^2) \text{Re } G'(M^2) \right] \\ G(p^2) &= \frac{p^4}{\pi} \int \frac{ds}{s(s-p^2)} \left(\frac{\lambda(s, m^2, \mu^2)}{s^2} \right)^{3/2}\end{aligned}\quad (38)$$

Thus the subtraction constants in (19) are:

$$A(0) = \frac{g_{a_1\pi\sigma}^2}{48\pi} \left[\text{Re } G(M^2) - M^2 \text{Re } G'(M^2) \right], \quad A'(0) = \frac{g_{a_1\pi\sigma}^2}{48\pi} \text{Re } G'(M^2) \quad (39)$$

The coupled propagators.

Loops:

$$\begin{aligned}J_{11}(p^2) &= g_1^2 \left[H(p^2) - H(\mu^2) - (p^2 - \mu^2) H'(\mu^2) \right] \equiv g_1^2 (p^2 - \mu^2)^2 H_2(p^2) \\ J_{12}(p^2) &= g_1 g_2 \left[H(p^2) - H(\mu^2) \right] \equiv g_1 g_2 (p^2 - \mu^2) H_1(p^2) \\ J_{22}^L(p^2) &= \tilde{M}^2 + g_2^2 H(p^2), \quad \tilde{M}^2 = M^2 + A(0) - g_2^2 H(0)\end{aligned}\quad (40)$$

where

$$H(p^2) = \frac{1}{\pi} \int \frac{ds}{s(s-p^2)} \left(\frac{\lambda(s, m^2, \mu^2)}{s^2} \right)^{1/2} \quad (41)$$

Full propagators are:

$$\begin{aligned} \Pi_{11}(p^2) &= \frac{1}{p^2 - \mu^2} \cdot \frac{\tilde{M}^2 + g_2^2 H(p^2)}{\tilde{D}(p^2)} \\ \Pi_{12}(p^2) &= \frac{-i g_1 g_2 H_1(p^2)}{\tilde{D}(p^2)} \\ \Pi_{22}^L(p^2) &= \frac{1 + g_1^2 (p^2 - \mu^2) H_2(p^2)}{\tilde{D}(p^2)}, \end{aligned} \quad (42)$$

where

$$D(p^2) = (p^2 - \mu^2) \tilde{D}(p^2)$$

Let us recall the notations:

$$g_1^2 = \frac{g_{\sigma\pi\pi}^2}{16\pi} \quad g_2^2 = (m^2 - \mu^2)^2 \cdot \frac{g_{a_1\pi\sigma}^2}{16\pi}$$

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